Statistics

Measures of Central Tendencies & Dispersion.

Series

↙ (Row) ↓ ↘

Individual series

marks:-	
1	35
2	40
3	32
4	27
5	35

Discrete series

x_i	f_i
23	7
25	8
28	4
40	3

continuous series

0-10	6
10-20	8
20-30	9
30-40	7

Inclusive series

x_i	f_i
L.L 1-10 U.L	2
11-20	7
21-30	8
31-40	4

→ Both Inclusive

[Distcrete Data]

exclusive series

x_i	f_i
0-10	2
10-20	7
20-30	8
30-40	4

→ upper limit excluded.

[continuous Data]

L.L → lower limit

U.L → upper limit

⇒ Discrete Data

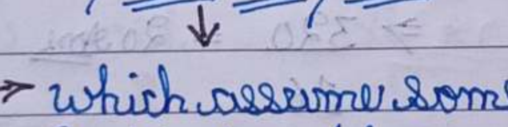
↓

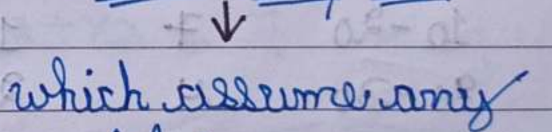
→ which assume some Particular Values.

⇒ continuous Data

↓

→ which assume any Value.





★ How to convert inclusive series into exclusive series

eg:- x_i f_i

x_i	f_i
1-10	2
11-20	7
21-30	8
31-40	4

⇒ LCL = D/2 ⇒ UCL = D/2

⇒

.5 - 10.5

10.5 - 20.5

20.5 - 30.5

30.5 - 40.5

LCB - UCB

Arithmetic mean

Type 1:-

- Individual series → $\bar{x} = \frac{\text{sum of obs}}{\text{Total No. of obs.}} = \frac{\Sigma x}{n}$

eg:- 2 7 9 14 28

$$\bar{x} = \frac{2+7+9+14+28}{5} = 12 \text{ Ans}$$

- Discrete series → $\frac{\Sigma f_i x_i}{\Sigma f_i}$

eg:-

x_i	f_i	$x_i f_i$
2	4	2x4
4	5	4x5
6	7	6x7
8	6	8x6
10	3	10x3

$$\Rightarrow \frac{148}{25} = 5.92 \text{ Ans}$$

- Continuous series → $\frac{\Sigma f_i x m_i}{f_i}$

eg:-

x_i	f_i	m_i	$m_i f_i$
0-10	2	5	5x2
10-20	7	15	15x7
20-30	4	25	25x4
30-40	3	35	35x3

$$\Rightarrow \frac{320}{16} = 20 \text{ Ans}$$

m.m.m Imp

Type-2 Base on Property → +/- → origin. → x/÷ → scale.

eg:-

x_i	$x_1 = x_i+2$	$x_2 = x_i-1$	$x_3 = 3x_i$
2	4	1	6
4	6	3	12
6	8	5	18
8	10	7	24
10	12	9	30

$\bar{x} = \frac{30}{5} = 6$ $\bar{x}_1 = \frac{40}{5} = 8$ $\bar{x}_2 = \frac{25}{5} = 5$ $\bar{x}_3 = \frac{90}{5} = 18$

$x_4 = x_i/2$	$x_5 = 3x+2$
1	8
2	14
3	20
4	26
5	32

$\bar{x}_4 = \frac{15}{5} = 3$ $\bar{x}_5 = \frac{100}{5} = 20$

$\boxed{y = ax + b}$ $\boxed{y = a + bx}$

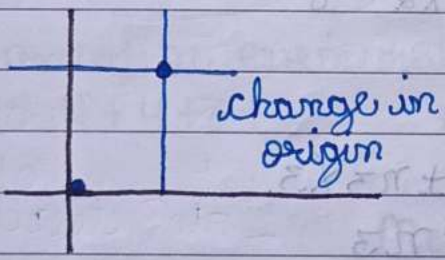
linear Reationship

$\Rightarrow \bar{y} = a\bar{x} + b$ $\Rightarrow \bar{y} = a + b\bar{x}$

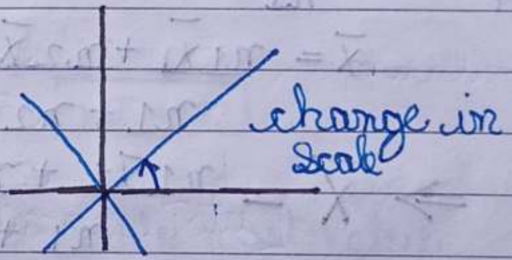
a = change of scale. b = change of scale

b = change of origin: a = change of origin

(i)



(ii)



(iii)

x_i	$x_i - \bar{x}$
2	-4
4	-2
6	0
8	2
10	4
$\bar{x} = 6$	0

$\Rightarrow \frac{x_1 + x_2 + x_3}{3} = \bar{x}$

$\rightarrow x_1 + x_2 + x_3 = 3\bar{x}$

$\bar{x} = \frac{\Sigma x}{n} \Rightarrow n\bar{x} = \Sigma x$

$\rightarrow x_1 - \bar{x} + x_2 - \bar{x} + x_3 - \bar{x}$

$x_1 + x_2 + x_3 - \bar{x} - \bar{x} - \bar{x}$

$3\bar{x} - 3\bar{x} = 0$

gap same

	I	D	C
D	$\frac{\Sigma x}{n}$	$\frac{\Sigma fx}{\Sigma f}$	$\frac{\Sigma fm}{\Sigma f}$ $m_i = \text{mid value} \frac{LCL+UCL}{2}$
S.C $d_i = x_i - A$	$\frac{\Sigma d}{n} + A$	$\frac{\Sigma fd}{\Sigma f} + A$	$\frac{\Sigma fd}{\Sigma f} + A$ $[d_i = m_i - A]$
S.D $u_i - \frac{x_i - A}{h}$ $hu_i + A_i = x_i$	$A + \frac{\Sigma u}{n} \times h$	$A + \frac{\Sigma fu}{\Sigma f} \times h$	$A + \frac{\Sigma fu}{\Sigma f} \times h$ $\left[u_i = \frac{m_i - A}{h}\right]$

Type-3 combined mean

eg:-

2 4 6 | 5 7

$n_1 = 3$ $\bar{x} = \frac{\Sigma x}{n_1}$ $\Sigma x = n_1 \bar{x}_1$ | $n_2 = 2$ $\Sigma x_2 = n_2 \bar{x}_2$

$\bar{x}_1 = 4$ | $\bar{x}_2 = 6$

$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2}{n_1 + n_2}$$

$$\Rightarrow \bar{X} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2 + n_3 \bar{x}_3}{n_1 + n_2 + n_3}$$

Type-4 Correct mean

eg:-

2 4 8 12 16 $n = 5$

$\Sigma x = 42$ $\bar{x} = 8.4$

$n = 5$ $\bar{x} = 8.4$ $IO = 12$ $CO = 10$

$$\Rightarrow \Sigma \bar{x} = n\bar{x} = 5 \times 8.4 = 42$$

$$\frac{42 - 12 + 10}{5} = \frac{40}{5} = 8$$

$$\Rightarrow \text{Correct Mean} = \frac{n\bar{x} - \text{Incorrect} + \text{correct}}{n}$$

★ Sum of n natural No's:-

$$1+2+3+\_\_\_+n = \boxed{\frac{n(n+1)}{2}}$$

★ Sum of Square of n natural No's:-

$$1^2+2^2+\_\_\_\_n^2 = \boxed{\frac{n(n+1)(2n+1)}{6}}$$

★ Sum of cubes of n natural No's:-

$$1^3+2^3+\_\_\_n^3 = \boxed{\left(\frac{n(n+1)}{2}\right)^2}$$

★ Sum of n odd natural No's:-

$$1+3+5+\_\_\_+(2n-1) = \boxed{n^2}$$

★ Sum of n even natural No's:-

$$2+4+6+\_\_\_\_+(2n) = \boxed{n(n+1)}$$

Median ⇒ middle most Value.

eg:- 8 17 2 7 10

ac → 2 7 ⑧ 10 17

dc → 17 10 ⑧ 7 2

$$\Rightarrow \left(\frac{n+1}{2}\right)^{th} \quad n=5$$

eg:- 2 7 8 10 17 20

$$\underbrace{8 \quad 10}_{\downarrow}$$

$$\frac{8+10}{2} = 9$$

$n=6$ $\quad \frac{n+1}{2}^{th} = \frac{6+1}{2} = \frac{7}{2}$ $\quad$ 3.5th obs.

→ 3rd + .5 [4th - 3rd]

n even median

$$\Rightarrow \frac{\frac{n}{2}^{th} + \left(\frac{n}{2}+1\right)^{th}}{2}$$

n odd median

$$\Rightarrow \left(\frac{n+1}{2}\right)^{th} \text{ obs.}$$

★ **Note** → To find out median first arrange data into ascending or Desending order.

· **Individual Series**:-

n odd → $\left(\frac{n+1}{2}\right)^{th}$ → Rank of median

n even → $\frac{\frac{n}{2}^{th} + \left(\frac{n}{2}+1\right)^{th}}{2}$ → $\frac{n+1}{2}$

Note:- single formula $\Rightarrow \left(\frac{n+1}{2}\right)^{th}$ obs..

eg:- 4.5 → $4^{th} + .5\,[5^{th} - 4^{th}]$

· **Discrete Series**:-

x_i f_i cf. 2 2 2 4 4 4 4 4

2 3+0=3 6 6 6 6 6 6 6 8

4 5+3=8 8 8 10 10

6 7+8=15

8 3+15=18

10 2+18=20

20

median.

$$\Rightarrow \left(\frac{20+1}{2}\right)^{th} = 10.5^{th}$$

$$6 + .5\,[0] = 6$$

Note:- First findout cf for Discrete series.

· **continuous Series**:-

$\left(\frac{N}{2}\right)^{th}$ item

$\Rightarrow l+\frac{\frac{N}{2}-f}{f}\times h$

xi	fi	cf
10-20	2	2
20-30	8	10 → median class
30-40	4	14
40-50	3	17
50-60	3	20

$\Rightarrow 20+\frac{10-2}{8}\times 10$

$20+\frac{\cancel{8}}{\cancel{8}}\times 10$

$\Rightarrow 30$

eg:-

xi	fi	cf
0-10	2	2
10-20	4	6
20-30	7	13
30-40	5	18
40-50	2	20

$$\Rightarrow \boxed{l+\frac{\frac{N}{2}-f}{f}\times h}$$

$\Rightarrow \frac{N}{2}=\frac{20}{2}=10^{th}$

$\Rightarrow 10-6=④$

eg:-

	xi	fi	cf
	100-200	7	7
	200-300	10	17
	300-400	15	32
M.C	400-500	18	50
	500-600	12	62
	600-700	10	72
		72	

$\text{Rank}=\frac{N}{2}\Rightarrow\frac{72}{2}=36$

18 — = 100

1 — = 100/18

4 — = 100/18 × 4

$\Rightarrow 22.2$

$\Rightarrow 400+22.\overline{2}$

$\Rightarrow 422.\overline{2}$

$\Rightarrow l+\frac{N/2-f}{f}\times h \Rightarrow 400+\frac{36-32}{18}\times 100$

$\Rightarrow 422.\overline{2}$ Ans

median class:-

l = lower limit.
cf = Previous cumulative frequency.
f = corresponding frequency.
h = class length.

Type-2 Based on properties

x_i	$+2$	-1	$\times 3$	$\div 2$
2	4	1	6	1
4	6	3	12	2
5	7	4	15	2.5
8	10	7	24	4
10	12	9	30	5
median = 5	$M = 7$	$M = 4$	$M = 4$	$M = 14.5$

$y = \underset{\substack{\downarrow \\ \text{c.o.o}}}{a} + \underset{\substack{\downarrow \\ \text{change of scale.}}}{b}x$

$y = \underset{\substack{\downarrow \\ \text{change of scale.}}}{a}x + b \hookrightarrow \text{c.o.o.}$

$y_{me} = a + b x_{me}$.

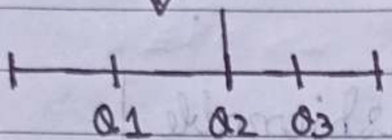
$y_{me} = a x_{me} + b$.

eg:- $y = 3 + 4x$ $\quad x_{me} = 5$

$y_{me} = 3 + 4 \times 5 = 23$

Type-3 Based on Partition Values

quartile → 4 equal Parts



Q_2 = median

Decile → 10 equal Parts → D_5 = median →

$$D_K = k\left(\frac{n+1}{10}\right)^{th}$$

Percentile → 100 equal Parts → P_{50} = median →

$$P_K = k\left(\frac{n+1}{100}\right)^{th}$$

I.S & D.S :-

Rank:- $Q_K = k\left(\frac{n+1}{4}\right)^{th}$

$$Q_2 = \cancel{2}\left(\frac{n+1}{\cancel{4}2}\right)$$

$$D_K = l + \frac{kN/10 - cf}{f} \times h$$

$$P_K = l + \frac{\frac{kn}{100} - f}{f} \times h$$

C.S :-

median

$$Q = Q_K = l + \frac{\frac{kn}{4} - f}{f} \times h$$

eg :-

xi	fi	cf
10-10	4	4
10-20	5	9
20-30	7	16
30-40	8	24
40-50	6	30
50-60	4	34
	34	

$\Rightarrow$ median $= 30 + \frac{1}{8} \times 10 = 31.25$

$\Rightarrow Q_1 = 10 + \frac{8.5-4}{5} \times 10 = 19$

$\Rightarrow Q_3 = 40 + \frac{25.5-24}{6} \times 10 \Rightarrow 42.5$

$\Rightarrow D_7 = 30 + \frac{23.8-16}{8} \times 10 \Rightarrow 39.75$

$\Rightarrow P_{23} = 10 + \frac{7.82-4}{5} \times 10 \Rightarrow 17.64$

Date ....

Type-1

[most Popular]

qualitative DataWork. **Mode** → Highest frequency.

eg:- $2 \quad 4 \quad \underbrace{8 \quad 8}_{\text{Mode}} \quad 7 \quad 14 \quad 17$

xi	fi
2	3
4	7
6	8
8	4

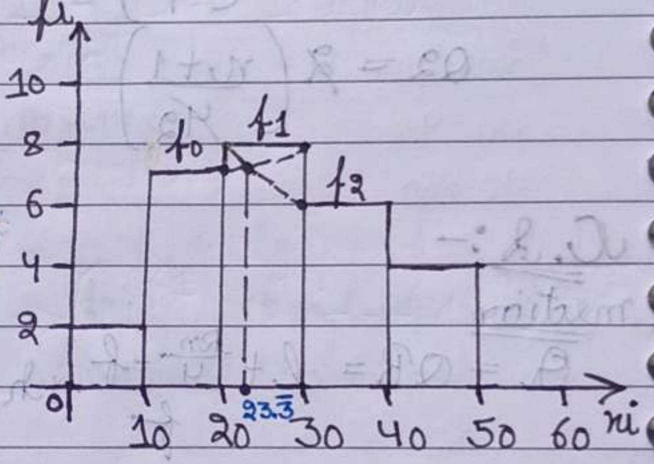
⇒ Mode is 6

[most fashionable
most Popular]

[mode find by graphically
↓
Histogram]

continous series:-

xi	fi	
0-10	2	
10-20	7	f_0
20-30	8	f_1 ⇒ modal class
30-40	6	f_2
40-50	4	



formula:- $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$$\Rightarrow 20 + \frac{8-7}{16-7-6} \times 10 \Rightarrow 20.\bar{3}$$

- ⇒ unimodal → 1 mode.
- ⇒ Bimodal → 2 mode.
- ⇒ Trimodal → 3 mode.
- ⇒ multimodal → more then 3.
- ⇒ Illdefined ←

$\Rightarrow \Delta 1 = f1 - f0$ $\Rightarrow f_1 + f_1 - f_0 - f_2$

$\rightarrow \Delta 2 = f1 - f0$

Dell ←

$(f1 - f0) + (f1 - f2)$

$$\boxed{\text{Same as} \Rightarrow l + \frac{\Delta 1}{\Delta 1 + \Delta 2} \times h}$$

Note:- To find out mode first convert inclusive series into exclusive.

Type-2 Based on Properties

$\Rightarrow y = a + b \cdot x$ $\Rightarrow y = ax + b$

$\Rightarrow y_{mo} = a + b \times mo$ $\Rightarrow y_{mo} = a \times mo + b$

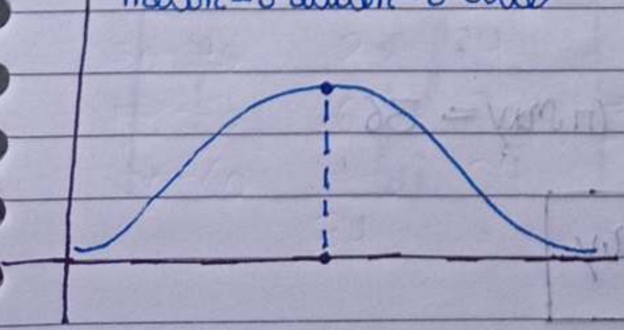
Type-3 Based on emphirical Relationships

$$\Rightarrow \underbrace{\text{mode} = 3\,\text{median} - 2\,\text{mean.}}_{\downarrow}$$

asymmetrical Distribution

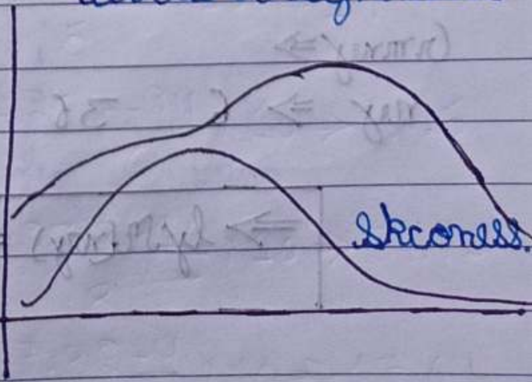
Symmetrical Distribution ↓

mean = median = mode



Asymmetrical Distribution [moderate] ↓

about to symmetric



Type-1

G.M → Geometric mean

Individual series:-

$$g.m = (x_1 \cdot x_2 \cdot ---- x_n)^{1/n}$$

eg:- 2 4 8.

$\Rightarrow (2.4.8)^{1/3} \Rightarrow (64)^{1/3} = 2^{\overset{2}{\cancel{6}} \times 1/\cancel{3}} \Rightarrow 2^2 = 4$

eg:- 4 8 10 15.

$\Rightarrow (4.8.10.15)^{1/4} \Rightarrow (4800)^{1/4} \Rightarrow 8.3235$

eg:- Type 2 8 10

$\Rightarrow (2.8.10)^{1/3} \Rightarrow (160)^{1/3} \Rightarrow 5.432$

calci:- 160 √ 12 times − 1 ÷ 3 + 1 = (x=) 12 times

Discrete series:-

x_i	f_i
2	2
4	3
6	2

$\Rightarrow (\underset{\sim}{2.2}.4.4.4.6.6)^{1/7}$

$(2^2 . 4^3 . 6^2)^{1/7} \Rightarrow (x_1^{f1} \; x_2^{f2} --- x_n^{fn})^{\frac{1}{\Sigma fi}}$

Type-2

Based on property

$x_i \Rightarrow$ 2 4 8 $\quad$ GMx = 4

$y_i \Rightarrow$ 3 9 27 $\quad$ GMy = 9

Gmxy ⇒

xy ⇒ 6 36 216 $\quad$ GMxy = 36

$$\boxed{\Rightarrow GM(xy) = GMx \times GMy}$$

$$\boxed{\Rightarrow GM\left(\frac{x}{y}\right) = \frac{GMx}{GMy}}$$

H.M ⇒ Harmonic mean

Type-1

· Individual series :-

eg :- 2 4 6

$$\Rightarrow \frac{3}{1/2 + 1/4 + 1/6}$$

$$\boxed{\Rightarrow \frac{n}{\Sigma \frac{1}{x_i}}}$$

calci :- 1÷2 M+ 1÷4 M+ 1÷6 M+

MRC (÷ = ×) 3 ⇒ 3.27

eg :- 5 8 12

$$\Rightarrow \frac{3}{1/5 + 1/8 + 1/12}$$

calci :- 1÷5 M+ 1÷8 M+ 1÷12 M+

MRC (÷ = ×) 3 ⇒ 7.34.

· Discrete Series :-

x_i	f_i
2	2
4	3
6	2
	7

$$\frac{7}{\underbrace{1/2 + 1/2 + 1/4 + 1/4 + 1/4 + 1/6 + 1/6}_{2 \cdot 1/2 + 3 \cdot 1/4 + 2 \cdot 1/6}} \Rightarrow 3.36$$

$$\boxed{\Rightarrow \frac{\Sigma f_i}{\Sigma \frac{f_1}{x_i}}} \quad \Rightarrow \frac{\Sigma f_i}{\frac{f_i}{x_i} + \frac{f_2}{x_2} -- \frac{f_n}{x_n}}$$

Type-2 Based on Relationship b/w A.M, G.M & H.M

eg:- $a \quad b$

$A.M = \frac{a+b}{2}$ $\quad G.M = (ab)^{1/2}$ $\quad H.M = \frac{2}{\frac{1}{a}+\frac{1}{b}} \downarrow 2ab/a+b$

$$\Rightarrow A.M \times H.M = \frac{a+b}{2} \cdot \frac{2ab}{a+b} = ab$$

$$\boxed{G.M^2 = A.M \times H.M}$$

eg:- $2 \quad 8$

$A.M = 5$ $\quad G.M = 4$ $\quad H.M = 3.2$

$$\boxed{A.M \geq G.M \geq H.M}$$

eg:- $4 \quad 4$ → [Constant/Same term]

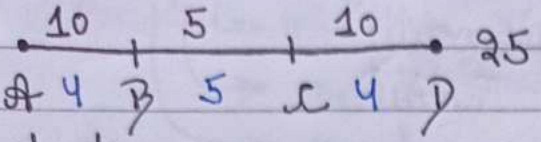
$A.M = 4$ $\quad G.M = 4$ $\quad H.M = 4$

$$\boxed{A.M = G.M = H.M}$$

eg:- terms Distinct (unequal)

$$\boxed{A.M > G.M > H.M}$$

Avg speed ⇒ H.M   **Avg. cost** $\boxed{H.M}$



speed:-

Avg. speed:- $\frac{\text{Total Distance}}{\text{Total Time}}$

Time:- $\frac{\text{Distance}}{\text{speed}}$

$$\Rightarrow \frac{10+5+10}{10/4+5/5+10/4} = 4.16$$

Distance must be same.

Type-3   **Combined H.M**

$$H.M = \frac{n}{\Sigma \frac{1}{x_i}} \qquad \Sigma \frac{1}{x_i} = \frac{n}{H}$$

$$\Sigma \frac{1}{x_i} = \frac{n_1}{H_1} \qquad \Sigma \frac{1}{x_2} = \frac{n_2}{H_2}$$

$$\boxed{\frac{n_1+n_2}{\frac{n_1}{H_1}+\frac{n_2}{H_2}}} \Rightarrow \text{Combined H.M}$$

Central Tendency

1. Rigidly Defined.
2. Based on all obs.
3. Affected by sampling fluctuation.
4. Possess some mathematical proper.
5. extreme obs.

Theory

* Properties:-

- → mean
- → median
- → mode

$y = a + bx$

a → origin (+/-)

b → scale (×/÷)

* mean ⇒ $\Sigma(x_i - \bar{x}) = 0$

$\Sigma(x_i - \bar{x})^2 = \text{minimum}$

$\Sigma(x_i - A) \neq 0$

$\Sigma(x_i - A) = 0$

A = Assumed Mean

A = Actual Mean

* median ⇒ $\Sigma|x_i - Me| = \text{minimum}$

Me → median

* combined :-

→ $A.M = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$

→ $G.M = (x_1^{f_1}\, x_2^{f_2} - - - x_n^{f_n})^{1/n_1}$

→ $H.M = \dfrac{n_1 + n_2}{\dfrac{n_1}{H_1} + \dfrac{n_2}{H_2}}$

* Based on all obs:- A.M / G.M / H.M.

* Graphically :-

↳ Partition Values.] → ogive

→ median.

→ mode → Histogram

ogive

less than Type   more than type

Median → one graph (curve) method

Median → Two graph (curve) method

- G.M use → Avg Rate or Avg Ratio or Avg Per.
- H.M use → Avg Rate or Avg Ratio or Avg speed or Avg cost.

G.M & H.M → Ratio Average.

	Mean	Median	Mode	G.M	H.M
Rigidly Defined	✓	✓	X	✓	✓
Based on all obs.	✓	X	X	✓	✓
Affected by sampling fluctuation	Very much	Not much	Not much	Not much	Not much
mathematical property	✓	X	X	✓	✓
Extreme obs..	Very much	No	No	Not much	Not much

Spiral

Date ....

⇒ For open & classification → median.
[Best measure of central tendency]

⇒ Best measure → mean
⇒ Most Popular → mode.
⇒ Difficult to comprehend & compute → G.M & H.M
⇒ qualitative Data → mode.

• Relation b/w. A.M / G.M / H.M :-

$G^2 = AH$

$$\begin{tikzcd}
 & A > G > H \arrow[dl] \arrow[dr] & \\
A = G = H & & A > G > H \\
\text{Constant} & & \text{Distient}
\end{tikzcd}$$

Dispersion

Absolute		Relative (unit free)
1. Range		1. coefficient of Range.
2. Q.D	quartile Deviation.	2. coefficient of Q.D.
3. M.D	mean Deviation.	3. coefficient of M.D.
4. S.D	standard Deviati..	4. coefficient of S.D.

Range

Type-1 Based on General formula:-

I.S → L−S

D.S →

x_i	f_i
2	4
4	7
6	8
8	3

$\Rightarrow 8-2=6$

C.S →

x_i	f_i
0−10	2
10−20	4
20−30	7
30−40	3

x_i	f_i
1−10	2
11−20	4
21−30	7
31−40	3

→ convert into inclusive

- Range = 40 − 0 = 40
- Range = 40.5 − .5 = 40

- coefficient of Range :- $\frac{L-S}{L+S} \times 100 \Rightarrow \frac{8-2}{8+2} \times 100 = 60$

Type 2:- Based on Property:-

I.S → 2 4 8 16 25

range = 25 − 2 = 23

x_i+2 → 4 6 10 18 27

range = 27 − 4 = 23

Date ....

$x_i - 1 \Rightarrow 1 \quad 3 \quad 7 \quad 15 \quad 24$

range $= 24 - 1 = 23$

$3x \Rightarrow 6 \quad 12 \quad 24 \quad 48 \quad 75$

range $= 75 - 6 = 69$

$x/2 \Rightarrow 1 \quad 2 \quad 4 \quad 8 \quad 25/2$

range $= 25/2 - 1 = 11.5$

$-3x \Rightarrow -6 \quad -12 \quad -24 \quad -48 \quad -75$

range $= -6 - (-75) = 69$

V.Imp

Range :- +/- origin No effect   X/ ÷ scale effect ✓

$\Rightarrow y = a + bx$ $\Rightarrow y = ax + b$

Rx = given, $\boxed{Ry = |b| Rx}$ Rx = given, $\boxed{Ry = |a| Rx}$

$\Rightarrow ax + by + c = 0$

Eg:- $3x + 2y + 5 = 0$ Rx given, $\boxed{Ry = \left|\frac{-a}{b}\right| Rx}$

$2y = -3x - 5$

$y = \frac{-3}{2}x - \frac{5}{2}$

Type-1 $M.D = \frac{\Sigma |x_i - \bar{x}|}{n}$

Discrete series:-

x_i	f_i	$x_i - \bar{x}$
2	2	3.1 ⇒ calci = 2-5.125 = 4= 6=8=
4	7	1.125
6	3	.875 $\bar{x} = 5.125$
8	4	2.875
	15	

Spiral

· **continuous series**:- $\bar{x} = 25.95$

x_i	f_i	m_i	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert m_i - \bar{x} \rvert$
0-10	2	5	20.95	41.9
10-20	5	15	10.95	54.75
20-30	7	25	0.95	6.65
30-40	3	35	9.05	27.15
40-50	4	45	19.05	76.2
	21			206.65

$$\Rightarrow \frac{206.65}{21} \Rightarrow 9.84$$

median = cts ⇒ cf $l + \frac{N/2 - f}{f} \times h$

midValue = mi ⇒ $\lvert m_i - \text{median} \rvert$
calculatio

Type-2:- Based on Property:-

$y = a + bx$ or $y = ax + b$

$m.D_x$ = given $\quad$ mD_x = given

$\Rightarrow m.D_y = \lvert b \rvert \, mD_x$ $\quad$ $mD_y = \lvert a \rvert \, mD_x$

$$\boxed{ax + by + c = 0}$$

mD_x = given

$$mD_y = \left\lvert -\frac{a}{b} \right\rvert mD_x$$

Type 3:- cofficient of m.D.

$$\Rightarrow \frac{\text{m.D about A}}{A} \times 100$$

A = mean or median or mode

quartite Deviation (Q.D)

$\downarrow$

Type-1 For open & classification Best Dispersion

$$\boxed{Q.D \Rightarrow \frac{Q_3 - Q_1}{2}} \rightarrow [\text{Semi-inter quartile Range}]$$

$$\Rightarrow 2Q.D = Q_3 - Q_1$$

Type-2 $ax + by + c = 0$

QD_x = given

$$QD_y = \left|\frac{-a}{b}\right| QD_x$$

$y = a + bx$

$Q.D_y = |b| Q.D_x$

$y = ax + b$

$QD_y = |a| QD_x$

Type-3 coefficient of Q.D

$$\Rightarrow \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100 \qquad \Rightarrow \frac{Q.D}{\text{median}} \times 100$$

$$\rightarrow \frac{\frac{Q_3 - Q_1}{2}}{\frac{Q_3 + Q_1}{100}} \times 100$$

Standard Deviation (S.D)

$$\boxed{S.D \Rightarrow \sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n}}}$$

Type 1 Individual series

$x_i = 2 \quad 5 \quad 8 \quad 4 \quad 7$

$\bar{x} = 5.2$

$(x_i-\bar{x}) = -3.2 \quad -0.2 \quad 2.8 \quad -1.2 \quad -1.8$

$(x_i-\bar{x})^2 = 2.1354$

calci ⇒ 3.2x = M+ 2.8x = M+ 1.8x = M+

0.2x = M+ 1.2x = M+ MR.C ÷ 5 = √

eg:- $x_i \Rightarrow 2 \quad 5 \quad 8 \quad 4 \quad 7 \quad \bar{x} = 5.2$

$x_i^2 \Rightarrow 4 \quad 25 \quad 64 \quad 16 \quad 49 = 158$

$$\Rightarrow \sqrt{\frac{\Sigma x_i^2}{n} - (\bar{x})^2} \Rightarrow \sqrt{\frac{158}{5} - (5.2)^2} \Rightarrow 2.135$$

Discrete:-

x_i	f_i	x_i^2
2	3	4
4	2	16
6	4	36
8	2	64

$$\Rightarrow \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$$

$$\Rightarrow \sqrt{\frac{\Sigma f_i x_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i x_i}{\Sigma f_i}\right)^2}$$

Shortcut method:-

$$d_i = x_i - A$$

$$\Rightarrow \sqrt{\frac{\Sigma f_i d_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i d_i}{\Sigma f_i}\right)^2}$$

- Step Deviation method :-

$$\Rightarrow u_i = \frac{x_i - A}{h}$$

$$\Rightarrow \sqrt{\frac{\Sigma f_i x_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)^2} \times h$$

Standard Deviation

Type:-1

Based on General formula

I.S $\Rightarrow \sqrt{\frac{\Sigma (x_i - \bar{x})^2}{n}}$ (or) $\sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$

(or) $\sqrt{\frac{\Sigma x_i^2}{n} - (\bar{x})^2}$

D.S $\Rightarrow \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}}$ (or) $\sqrt{\frac{\Sigma f_i x_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i x_i}{\Sigma f_i}\right)^2}$

(or) $\sqrt{\frac{\Sigma f_i x_i^2}{\Sigma f_i} - (\bar{x})^2}$

C.S $\Rightarrow x_i = m_i$

$u_i \Rightarrow \frac{x_i - A}{h}$

S.C

$D_i = x_i - A$

$$\Rightarrow \sqrt{\frac{\Sigma f_i d_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i d_i}{\Sigma f_i}\right)^2}$$

S.D $\Rightarrow$

$$\Rightarrow \sqrt{\frac{\Sigma f_i u_i^2}{\Sigma f_i} - \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)^2} \times h$$

Type:-2 Variance = $(S.D)^2$

Type:-3 **Based on Property**

$y = a + bx$
$SDx = \text{given}$
$SDy = |b|\, SDx$

$y = ax + b$
$SDx = \text{given}$
$SDy = |a|\, SDx$

$ax + by + c = 0$ SDx given

$$\Rightarrow SDy = \left|\frac{-a}{b}\right| SDx$$

⇒ **coefficient of Variation** :- $\boxed{\frac{\sigma}{\bar{x}} \times 100}$

$$C.V \Rightarrow \frac{S.D}{\text{mean}} \times 100$$

Imp (xi) S.D

⇒ n natural No's ⇒

1 2 3 ..... n

$$S.D = \sqrt{\frac{\Sigma xi^2}{n} - \left(\frac{\Sigma xi}{n}\right)^2} \qquad \Sigma xi = \frac{n(n+1)}{2}$$

$$\Rightarrow \sqrt{\frac{\cancel{n}(n+1)(2n+1)}{6\cancel{n}} - \frac{\cancel{n^2}(n+1)^2}{4.\cancel{n^2}}}$$

$$\Rightarrow \sqrt{\frac{n+1}{2}\left[\frac{2n+1}{3} - \frac{n+1}{2}\right]} \Rightarrow \sqrt{\frac{n+1}{2}\left[\frac{4n+2-3n-3}{6}\right]}$$

$$\Rightarrow \sqrt{\frac{n+1}{2} \cdot \frac{n-1}{6}} = \sqrt{\frac{n^2-1}{12}}$$

* S.D of n natural No's ⇒ $\sqrt{\frac{n^2-1}{12}}$

Variance of n natural No's ⇒ $\frac{n^2-1}{12}$

Ideal measure of Dispersion

1. Rigidaly Defined.
2. Based on all obs.
3. effect of extreme obs.
4. sampling fluctuation.
5. Posses some mathematical Property.

MIND MAP

RITU JINDAL

MEASURES OF CENTRAL TENDENCY

Arithmetic Mean

Individual Series

- **Direct** ⇓ $\overline{X} = \frac{\Sigma x}{n}$
- **Shortcut** ⇓ $\overline{X} = A + \frac{\Sigma d}{n}$, $d_i = x_i - A$
- **Step Deviation** ⇓ $\overline{X} = A + \frac{\Sigma u}{n} \times h$, $u = \frac{x_i - A}{h}$

Discrete Series

- **Direct** ⇓ $\overline{X} = \frac{\Sigma f x}{\Sigma f}$
- **Shortcut** ⇓ $\overline{X} = A + \frac{\Sigma fd}{\Sigma f}$, $d = x_i - A$
- **Step Deviation** ⇓ $\overline{X} = A + \frac{\Sigma fu}{\Sigma f} \times h$, $u_i = \frac{x_i - A}{h}$

Correcting AM

⇓

$$\frac{\text{Incorrect Sum} - \text{Incorrect } x + \text{Correct } x}{n}$$

Important Points

⇓

Best Measure
Most Commonly Used

Continuous Series

- **Direct** ⇓ $\overline{X} = \frac{\Sigma fm}{\Sigma f}$, m = mid Point
- **Shortcut** ⇓ $\overline{X} = A + \frac{\Sigma fd}{\Sigma fi}$, $d_i = m_i - A$
- **Step Deviation** ⇓ $\overline{X} = A + \frac{\Sigma fu}{\Sigma f} \times h$, $u_i = \frac{m_i - A}{h}$

Some Important formulae

⇓

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Combined A.M.

$$\overline{X} = \frac{n_1\overline{x}_1 + n_2\overline{x}_2}{n_1 + n_2}$$

$$\overline{X} = \frac{n_1\overline{x}_1 + n_2\overline{x}_2 + n_3\overline{x}_3}{n_1 + n_2 + n_3}$$

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MIND MAP

MEASURES OF CENTRAL TENDENCY

Median

- Individual Series
  - Arrange Data in ascending or Descending order
    - Odd ⇒ $\left(\frac{n+1}{2}\right)^{th}$ item
    - Even ⇒ $\frac{\left(\frac{n}{2}\right)^{th}+\left(\frac{n}{2}+1\right)^{th}}{2}$
- Discrete Series
  - Arrange Data in ascending or Descending order
  - ⇓ Find out Cumulative frequency
  - ⇓ $\left(\frac{n+1}{2}\right)^{th}$ obs.
- Continuous Series
  - Arrange Data in Ascending or Descending order
  - ⇓ Find out Cumulative frequency
  - ⇓ $M = l + \frac{\frac{N}{2} - cf}{f} \times h$

Properties of Median

- $Y = a + bx$
  $Y_{me} = a + bX_{me}$
- $\Sigma|x_i - \text{median}| = \text{minimum}$

For open-end Classification ⇒ Median is best

Mode

- Individual ⇓ Highest frequency
- Discrete ⇓ Highest frequency
- Continuous ⇓ $l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

Property of Mode → $Y = a + bx$
$Y_{mo} = a + bX_{mo}$

Partition Values

- Quartile
- Decile
- Percentile

Arrange Data Into Ascending Order Only

for Individual and Discrete ⇒
$Q_k = k\left(\frac{n+1}{4}\right)^{th}$ $D_k = k\left(\frac{n+1}{10}\right)^{th}$
$P_k = k\left(\frac{n+1}{100}\right)^{th}$

for Continuous Series ⇒
$Q_k = l + \frac{k\left(\frac{N}{4}\right) - cf}{f} \times h$
$D_k = l + \frac{k\left(\frac{N}{10}\right) - cf}{f} \times k$
$P_k = l + \frac{k\left(\frac{N}{100}\right) - cf}{f} \times k$

Emphetical Relation

⇓
Mode = 3 median − 2 Mean
Mean − Mode = 3(Mean − Median)

MIND MAP

MEASURES OF CENTRAL TENDENCY

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Geometric Mean

- Individual ⇓ $(x_1 \cdot x_2 \cdots x_n)^{\frac{1}{n}}$
- Discrete ⇓ $(x_1^{f_1} \cdot x_2^{f_2} \cdots x_n^{f_n})^{\frac{1}{\Sigma f_i}}$
- Continuous ⇓ $(m_1^{f_1} \cdot m_2^{f_2} \cdots m_n^{f_n})^{\frac{1}{\Sigma f_i}}$

G.M. Logarithmic formulae

⇓

Antilog $\frac{\Sigma \log x}{N}$

Combined G.M.

⇓

$(x_{11}\, x_{12} \cdots x_{1n_1}\, x_{21}\, x_{22} \cdots x_{2n_2})^{\frac{1}{n_1+n_2}}$

Harmonic Mean

- Individual ⇓ $\frac{n}{\frac{1}{x_1}+\frac{1}{x_2}+\cdots+\frac{1}{x_n}}$
- Discrete ⇓ $\frac{\Sigma f}{\frac{f_1}{x_1}+\frac{f_2}{x_2}+\cdots+\frac{f_n}{x_n}}$
- Continuous ⇓ $\frac{\Sigma f}{\frac{f_1}{m_1}+\frac{f_2}{m_2}+\cdots+\frac{f_n}{m_n}}$ m_i = mid-value

Combined H.M.

⇓

$$H = \frac{n_1 + n_2}{\frac{n_1}{H_1} + \frac{n_2}{H_2}}$$

Relationship Between A.M G.M. H.M

⇓

(i) $A \geq G \geq H$ (No indication)

$A > G > H$ (Unequal)

$A = G = H$ (Equal)

(ii) $A = \frac{a+b}{2}$ $G = \sqrt{ab}$

$H = \frac{2ab}{a+b}$

(ii) $G^2 = AH$

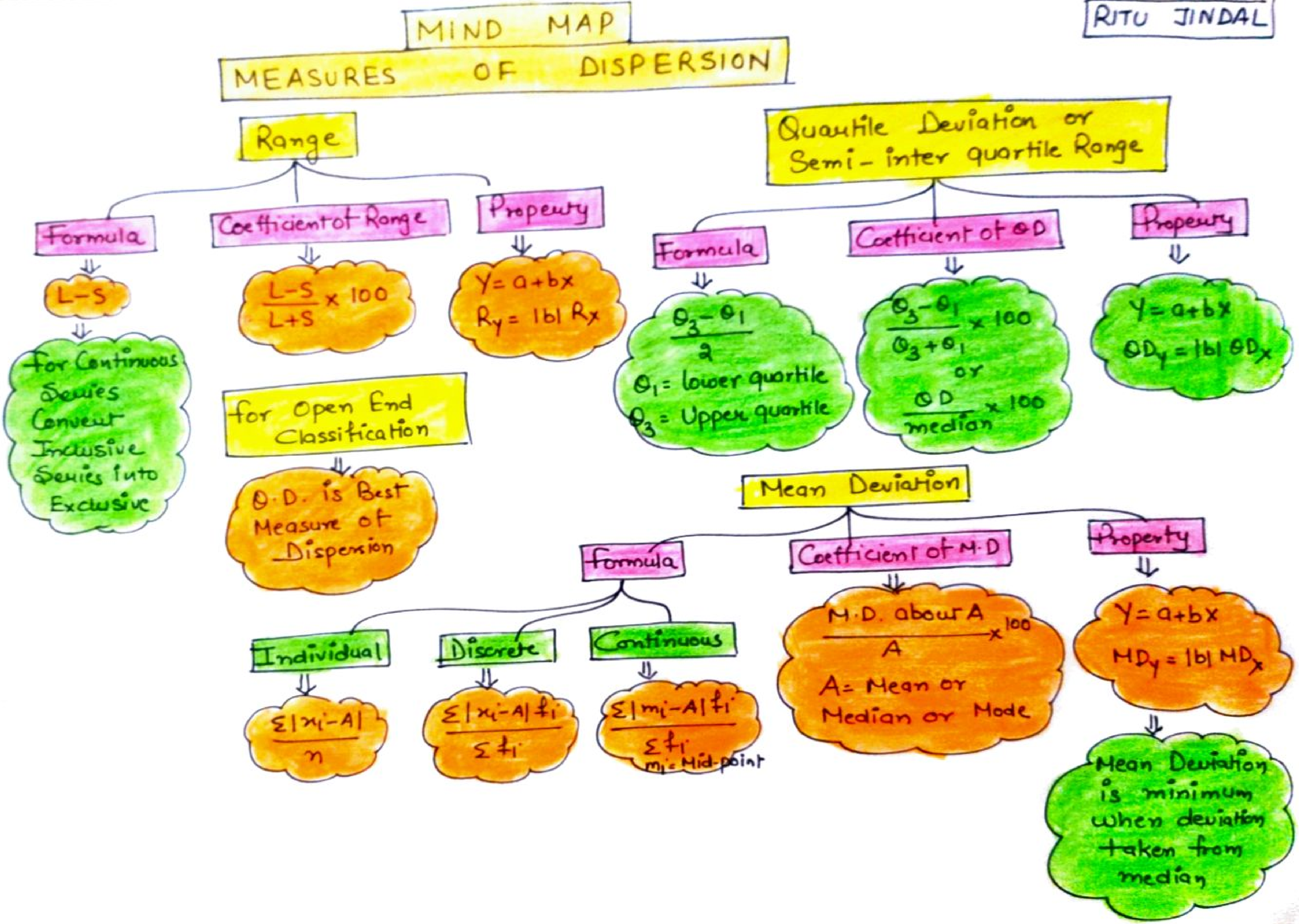
Ratio Avg

- G.M.
- H.M.

Avg. Rate, Ratio, Percentage → G.M.

Avg. Rate and Ratio → H.M.

Avg. Speed or Avg Cost → H.M.



MIND MAP

MEASURES OF DISPERSION

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Range

- Formula
  - $L-S$
  - For Continuous Series Convert Inclusive Series into Exclusive
- Coefficient of Range
  - $\frac{L-S}{L+S} \times 100$
- Property
  - $Y = a+bx$
  - $R_y = |b| R_x$

for Open End Classification

- Q.D. is Best Measure of Dispersion

Quartile Deviation or Semi-inter quartile Range

- Formula
  - $\frac{Q_3-Q_1}{2}$
  - Q_1 = lower quartile
  - Q_3 = Upper quartile
- Coefficient of QD
  - $\frac{Q_3-Q_1}{Q_3+Q_1} \times 100$
  - or
  - $\frac{QD}{median} \times 100$
- Property
  - $Y = a+bx$
  - $QD_y = |b| QD_x$

Mean Deviation

- Formula
  - Individual
    - $\frac{\Sigma|x_i-A|}{n}$
  - Discrete
    - $\frac{\Sigma|x_i-A|f_i}{\Sigma f_i}$
  - Continuous
    - $\frac{\Sigma|m_i-A|f_i}{\Sigma f_i}$
    - m_i = Mid-point
- Coefficient of M.D
  - $\frac{M.D. \text{ about } A}{A} \times 100$
  - A = Mean or Median or Mode
- Property
  - $Y = a+bx$
  - $MD_y = |b| MD_x$
  - Mean Deviation is minimum when deviation taken from median

MIND MAP

MEASURES OF DISPERSION

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Standard Deviation

Individual Series

$$\sqrt{\frac{\Sigma(x_i-\bar{x})^2}{n}}$$

$$\sqrt{\frac{\Sigma x_i^2}{n}-(\bar{x})^2}$$

Discrete Series

$$\sqrt{\frac{\Sigma f_i(x_i-\bar{x})^2}{\Sigma f_i}}$$

$$\sqrt{\frac{\Sigma f_i x_i^2}{\Sigma f_i}-\left(\frac{\Sigma f_i x_i}{\Sigma f_i}\right)^2}$$

or

$$\sqrt{\frac{\Sigma f_i x_i^2}{\Sigma f_i}-(\bar{x})^2}$$

Continuous Series

$$\sqrt{\frac{\Sigma f_i(m_i-\bar{x})^2}{\Sigma f_i}}$$

$$\sqrt{\frac{\Sigma f_i m_i^2}{\Sigma f_i}-\left(\frac{\Sigma f_i m_i}{\Sigma f_i}\right)^2}$$

or

$$\sqrt{\frac{\Sigma f_i m_i^2}{\Sigma f_i}-(\bar{x})^2}$$

Shortcut Method

$$\sqrt{\frac{\Sigma f_i d_i^2}{\Sigma f_i}-\left(\frac{\Sigma f_i d_i}{\Sigma f_i}\right)^2}$$

Step Deviation Method

$$\sqrt{\frac{\Sigma f_i d_i'^2}{\Sigma f_i}-\left(\frac{\Sigma f_i u_i'}{\Sigma f_i}\right)^2} \times h$$

Variance $= S.D^2$

$C.V = \frac{\sigma}{\bar{x}} \times 100$

$Y = a + bx$

$SD_y = |b| SD_x$

Best Measure of Dispersion

⇓

S.D.

S.D. of n natural No.

$$\sqrt{\frac{n^2-1}{12}}$$

⇓

SD of 2 Item $= \frac{|a-b|}{2}$

Combined S.D.

$$S = \sqrt{\frac{n_1 s_1^2 + n_1 s_2^2 + n_1 d_1^2 + n_2 d_2^2}{n_1+n_2}}$$

$d_1 = \bar{x}_1 - \bar{x}$

$d_2 = \bar{x}_2 - \bar{x}$

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2}$$

Correct S.D.

find Summation terms

Correct them

Put in formula

Relation

QD : MD : SD

10 : 12 : 15

QD<MD<SD